Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Tuesday 3 November 2020 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

8820 – 7301

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

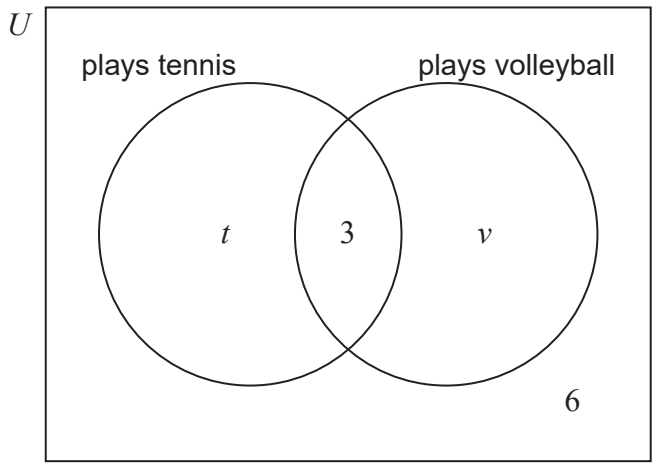
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

In a class of 30 students, 19 play tennis, 3 play both tennis and volleyball, and 6 do not play either sport.

The following Venn diagram shows the events "plays tennis" and "plays volleyball". The values t and v represent numbers of students.



(a) (i) Find the value of t.

(ii) Find the value of v. [4]

(b) Find the probability that a randomly selected student from the class plays tennis or volleyball, but not both. [2]

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(Question 1 continued)

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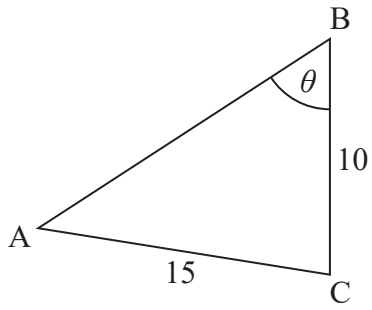
2. [Maximum mark: 6]

The following diagram shows a triangle ABC.

diagram not to scale



$AC = 15\,\text{cm}$, $BC = 10\,\text{cm}$, and $A\hat{B}C = \theta$.

Let $\sin C\hat{A}B = \frac{\sqrt{3}}{3}$.

(a) Given that $A\hat{B}C$ is acute, find $\sin\theta$. [3]

(b) Find $\cos(2 \times C\hat{A}B)$. [3]

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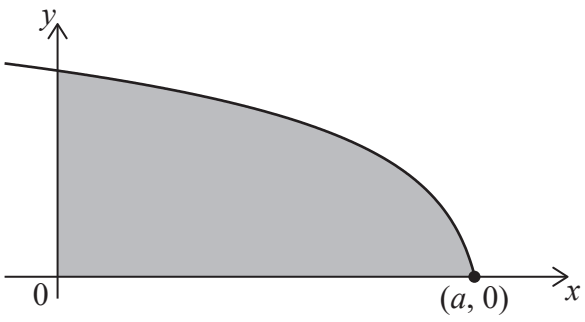
3. [Maximum mark: 7]

Let $f(x) = \sqrt{12 - 2x}$, $x \leq a$. The following diagram shows part of the graph of f.

The shaded region is enclosed by the graph of f, the x-axis and the y-axis.



The graph of f intersects the x-axis at the point $(a, 0)$.

(a) Find the value of a. [2]

(b) Find the volume of the solid formed when the shaded region is revolved 360° about the x-axis. [5]

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4. [Maximum mark: 6]

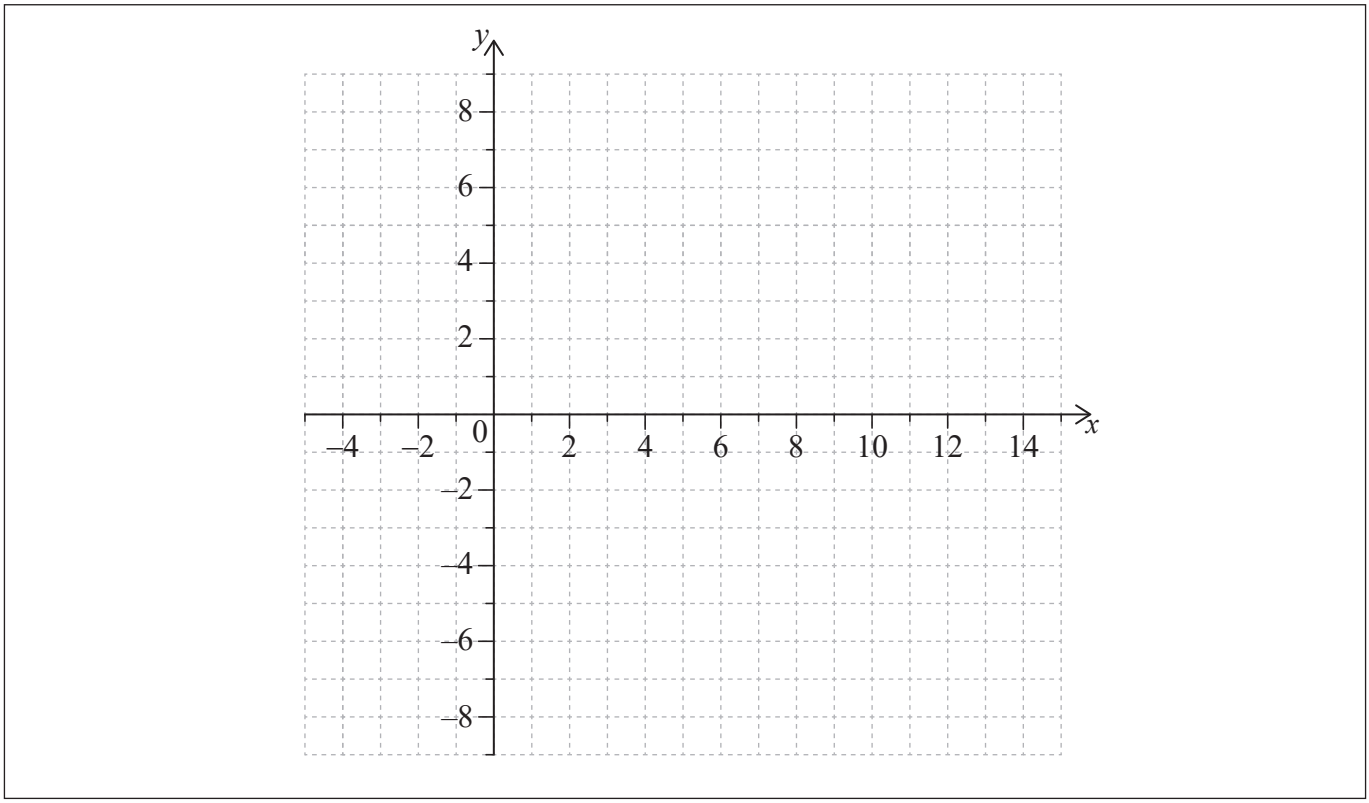
Let $f(x) = a\log_3(x-4)$, for $x > 4$, where $a > 0$.

Point $\mathrm{A}(13, 7)$ lies on the graph of f.

(a) Find the value of a. [3]

The x-intercept of the graph of f is $(5, 0)$.

(b) On the following grid, sketch the graph of f. [3]



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(Question 4 continued)

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will not be marked.

5. [Maximum mark: 6]

Let $f(x) = -x^2 + 4x + 5$ and $g(x) = -f(x) + k$.

Find the values of k so that $g(x) = 0$ has no real roots.

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6. [Maximum mark: 7]

The graph of a function f passes through the point $(\ln 4, 20)$.

Given that $f'(x) = 6e^{2x}$, find $f(x)$.

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7. [Maximum mark: 7]

In this question, all lengths are in metres and time is in seconds.

Consider two particles, P_1 and P_2, which start to move at the same time.

Particle P_1 moves in a straight line such that its displacement from a fixed-point is given by $s(t) = 10 - \frac{7}{4}t^2$, for $t \geq 0$.

(a) Find an expression for the velocity of P_1 at time t. [2]

Particle P_2 also moves in a straight line. The position of P_2 is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

The speed of P_1 is greater than the speed of P_2 when $t > q$.

(b) Find the value of q. [5]

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Section B

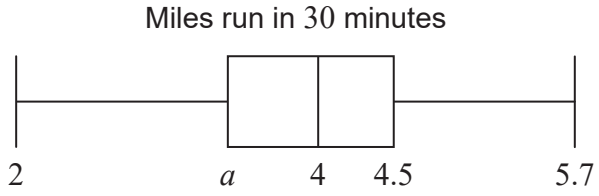
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Each athlete on a running team recorded the distance (M miles) they ran in 30 minutes.

The median distance is 4 miles and the interquartile range is 1.1 miles.

This information is shown in the following box-and-whisker plot.



(a) Find the value of a. [2]

The distance in miles, M, can be converted to the distance in kilometres, K, using the formula $K = \frac{8}{5} M$.

(b) Write down the value of the median distance in kilometres (km). [1]

The variance of the distances run by the athletes is $\frac{16}{9}\text{km}^2$.

The standard deviation of the distances is b miles.

(c) Find the value of b. [4]

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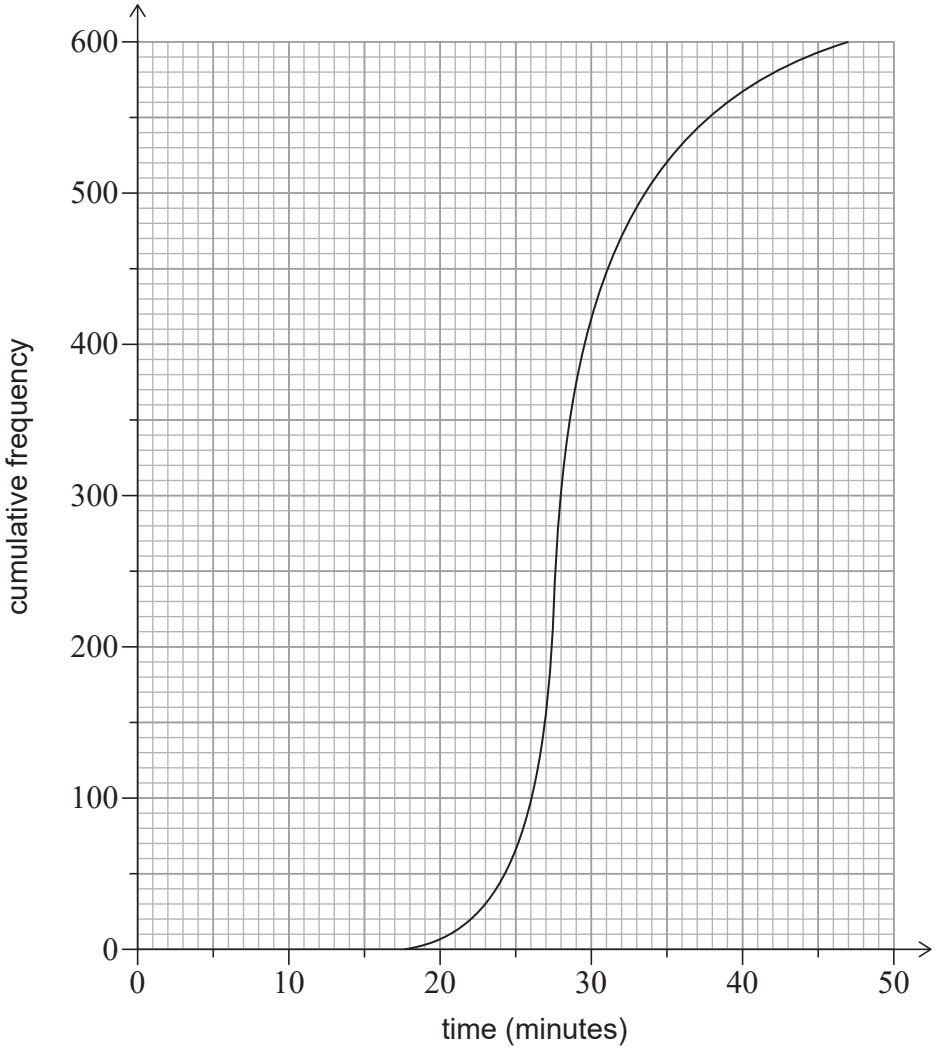
(Question 8 continued)

A total of 600 athletes from different teams compete in a $5\,\text{km}$ race. The times the 600 athletes took to run the $5\,\text{km}$ race are shown in the following cumulative frequency graph.



There were 400 athletes who took between 22 and m minutes to complete the $5\,\text{km}$ race.

(d) Find m. [3]

The first 150 athletes that completed the race won a prize.

(e) Given that an athlete took between 22 and m minutes to complete the $5\,\text{km}$ race, calculate the probability that they won a prize. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

Points A and B have coordinates $(1, 1, 2)$ and $(9, m, -6)$ respectively.

(a) Express $\vec{AB}$ in terms of m. [2]

The line L, which passes through B, has equation $\boldsymbol{r} = \begin{pmatrix} -3 \\ -19 \\ 24 \end{pmatrix} + s\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$.

(b) Find the value of m. [5]

Consider a unit vector $\boldsymbol{u}$, such that $\boldsymbol{u} = p\boldsymbol{i} - \frac{2}{3}\boldsymbol{j} + \frac{1}{3}\boldsymbol{k}$, where $p > 0$.

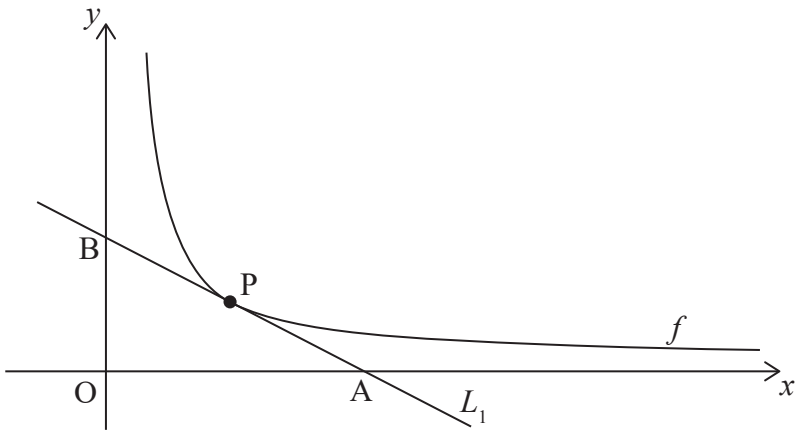
Point C is such that $\vec{BC} = 9\boldsymbol{u}$.

(c) Find the coordinates of C. [8]

10. [Maximum mark: 15]

The following diagram shows part of the graph of $f(x) = \frac{k}{x}$, for $x > 0$, $k > 0$.

Let $P\left(p, \frac{k}{p}\right)$ be any point on the graph of f. Line L_1 is the tangent to the graph of f at P.



(a) (i) Find $f'(p)$ in terms of k and p.

(ii) Show that the equation of L_1 is $kx + p^2y - 2pk = 0$. [4]

Line L_1 intersects the x-axis at point $A(2p, 0)$ and the y-axis at point B.

(b) Find the area of triangle AOB in terms of k. [5]

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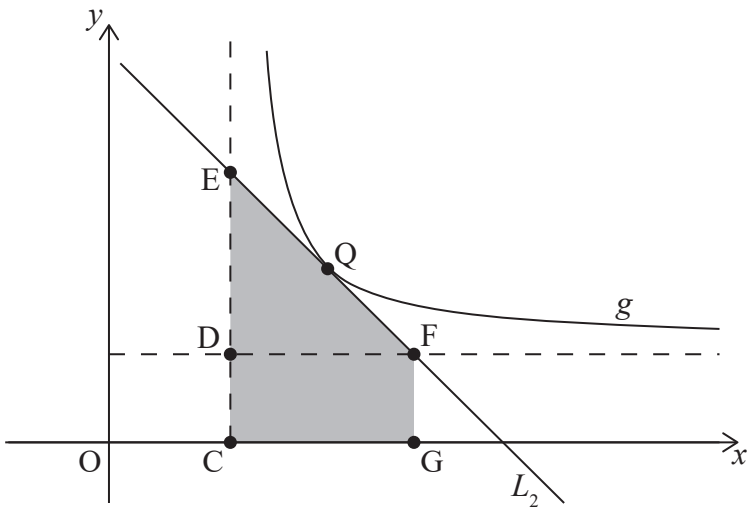
(Question 10 continued)

The graph of f is translated by $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ to give the graph of g.

In the following diagram:

- point Q lies on the graph of g
- points C, D and E lie on the vertical asymptote of g
- points D and F lie on the horizontal asymptote of g
- point G lies on the x-axis such that FG is parallel to DC.

Line L_2 is the tangent to the graph of g at Q, and passes through E and F.



(c) Given that triangle EDF and rectangle CDFG have equal areas, find the gradient of L_2 in terms of p. [6]

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will not be marked.